Assessing the Fractal Dimension and the Normalized Multiscale Bending Energy for Applications in Neuromorphometry

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Abstract. This work reports about the assessment of two shape descriptors, namely the fractal dimension and the normalized multiscale bending energy, for applications in biological shape analysis. Both descriptors are briefly reviewed and some preliminary results of their application to the morphometric characterization of neural cells are presented. The methodology for the evaluation of the shape measures, which is currently being carried out, is explained.

1 Introduction

In spite of the fact that human beings can readily recognize a multitude of known shapes, and even describe unknown objects based on analogies, shape analysis has turned out to be a difficult task to be implemented by automatic means. The main causes of such difficulties are related to the poor knowledge on the actual shape analysis mechanisms employed by biological systems, as well as the limited performance of the existing computational shape theories with respect to general and real data conditions, such as geometric variations and noise. The ability to correctly characterize shapes has become particularly important in biological and biomedical sciences, where morphological information about the specimen of interest can be used in a number of different ways such as for taxonomic classification and research on morphology-function relationships. It is thus important to define good shape measures (SM's) that can be effectively applied to biological shapes, so that they can be compared and analyzed by meaningful and objective criteria. The definition of such a set of SM's should pass through a thorough evaluation/assessment of their performance under different conditions. This work reports about ongoing efforts towards the evaluation of two kinds of SM's, namely the fractal measures-based [1] and the normalized multiscale bending energy (NMBE) [2], for neuromorphometry, i.e. morphological characterization of neural cells and structures. These two SM's, which had already been separately analyzed in previous works [2, 1], are currently being assessed under a more complete framework, which includes their use for the difficult task of automatic neural cell classification based on their shape. This work starts by revising the main concepts related to the fractal dimension estimation methods, including a discussion about the problems of limited fractality and spatial quantization of computer images, as well as the features of the NMBE in Section 2. Some preliminary results with respect to the application of these SM's to neural shapes are presented. Section 3 discusses the assessment methodology. The text concludes with some comments on the research that is being currently pursued.

2 Fractal Measures and the Normalized Multiscale Bending Energy

The Box-Counting Dimension: The typical technique for determination of the box-counting dimension (BCD) consists in partitioning the image space in boxes of size $d \times d$ and counting the number $N(d)$ of boxes that contain at least one part of the shape to be investigated. Several values of d are chosen and the least square linear fitting of $\log(N(d)) \times \log(d)$ is used to determine the value of D. However, this approximation will suffer the effects caused by spatial quantization as well as the limited fractality of most natural objects (such as neurons). As d becomes very small, the portions of the contours with limited fractality inside the boxes will approach a straight line segment, implying the fractal dimension (FD) of the curve to be $D = 1$. Therefore the curve $\log(N(d)) \times \log(d)$ will exhibit two distinct regions:

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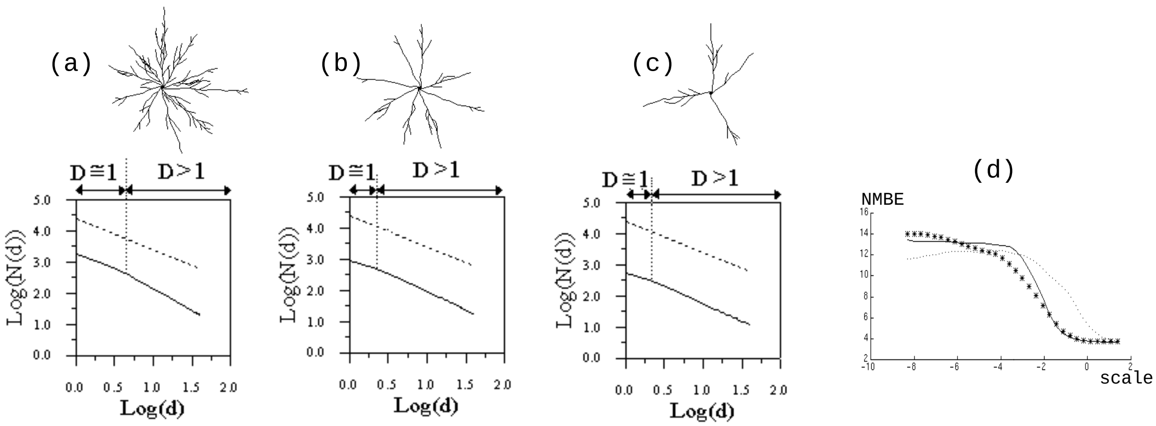

Figure 1: **(a)-(c)** Neural cells and their respective curves $\log(N(d)) \times \log(d)$; **(d)** NMBE (log-log plot) for the shapes (b) ("$*$"), (c) ("–") and (d) ("...").

one where the curve is fractal ($D > 1$) and one where it is not, as showed in Figure 1 (a)-(c) [1]. If we calculate D in the region where $D > 1$, the errors are thus minimized. We applied such guidelines to neural cells and obtained the results showed in Figure 1 (a),(b) and (c), where each cell is presented with its respective curve. The measured fractal dimension of these cells are 1.38, 1.24 and 1.12, respectively. Two regions can be identified from such curves in such a way that the more the cell 'fill in the space', the more inclined becomes the curve $\log(N(d)) \times \log(d)$, thus implying higher FD.

Curvegrams and Estimation of the Bending Energy: The NMBE is calculated from the multiscale curvature description of the initial contour, namely the *curvegram* $k(\sigma, t)$ [3]. The curvegram is a representation defined from a family of curves evolving as a function of a scale parameter σ. The curve evolution is controlled by filtering the original curve with a bank of Gaussian filters with bandwidth $1/\sigma$. It is well known that Gaussian low-pass filtering affect the power spectrum of the signal, leading to the undesirable effect of shrinking the contour. In the NMBE shape analysis framework, this problem has been circumvented by a perimeter preserving compensation scheme introduced in [2]. The curvegram $k(\sigma, t)$ describes the curvature function of the contour $c(t)$ smoothed by a filter of bandwidth $\tau = 1/\sigma$. The curve evolution $c(\sigma, t)$ is constrained so that all curves have the same perimeter as the original curve. Scaling invariance is obtained by normalizing the NMBE as a function of the perimeter $\mathcal{L}$ of the original curve. This normalization gives rise to the NMBE $\Psi(\sigma)$, which is defined by $\Psi(\sigma) = (\mathcal{L}^2/N)\sum_{t=0}^{N-1} k(\sigma, t)^2$. The NMBE for the artificial neural cells presented in Figure 1 (a), (b) and (c) are shown in Figure 1 (d).

3 Performance Assessment Methodology

The above discussed SM's are being assessed with respect to the following two special criteria: discrimination capabilities and robustness. The discrimination criterion concerns the performance of the SM's when dealing with similar shapes. The robustness criterion relates to the performance of the SM's under typical variations typically found in image-based measurement systems such as image acquisition differences (e.g. illumination conditions for outdoor cameras or differences in microscopy's focus), geometrical transformations, quantization and noise. The evaluation of the SM's will be carried out in two different ways, which also define two practical applications of such features. First, the SM's results are used for computer-aided analysis of neural cells, where the SM's results are judged against results produced by human visual inspection. In a second step, the problem of automatic classification of neural cells based on SM's features will be addressed. These topics are currently being developed and new results will be reported opportunely.

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